Earth Co-orbital Asteroid 2002 AA29

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Abstract

Asteroid 2002 AA29 is one of three known examples of horseshoe orbit behaviour. The asteroid is co-orbital with Earth; the only substantial difference in orbital parameters being a 10.7° inclination. From time to time, 2002 AA29 leaves the horseshoe behaviour and enters a quasi-satellite relationship with Earth (Connors et al. [1]), the next instance being approximately 2580 AD. Using the SWIFT simulation suite running on the Swinburne supercomputer this project investigates the orbital behaviour of 2002 AA29, in particular the quasi-satellite phase. The investigations confirm the presence of quasi-satellite behaviour and discount the possibility of resonances with Jupiter causing the transition.

1 Introduction

The solar system consists of many thousands of objects orbiting the Sun and planets. Between these objects are many gravitational interactions. Some are obvious, keeping planets and moons in orbit, while others are far more subtle, accounting for gaps in the main asteroid belt or the complex beauty of Saturn's rings. Among approximately 2200 near-Earth objects [2] are asteroids 3753 Cruithne and 2002 AA29, with orbital periods commensurate to Earth's and orbits that show particularly interesting dynamics. Both asteroids show orbital behaviour called a horseshoe orbit but only 2002 AA29 has an orbit similar to that of Earth. 2002 AA29 qualifies as co-orbital with Earth on the basis that it "shares the same orbit with a larger perturber" [3]. Analysis by Connors, Chodas, Mikkola, Wiegert, Veillet, and Innanen [1] indicates that AA 29's orbit has periods where it deviates from the horseshoe pattern and becomes a quasi-satellite of Earth. The quasi-satellite phase of 2002 AA29's orbit forms the subject of this project.

2 Background

The equations of motion for gravitational systems with three or more bodies are not generally analytically integrable. There exist, however, a number of tractable three-body restricted cases for which motions have been determined by mathematicians such as Euler, Lagrange, Jacobi, and Poincaré. Examination of such systems in a frame that co-rotates with the planet led to the derivation of Lagrangian equilibrium points (designated L1 through L5) in two-body systems at which a third, relatively light object may be dynamically stable. The light object may be seen as trapped in a gravitational well bounded by high walls near the planet, primary troughs at the L4 and L5 Lagrange points, and a secondary point at superior conjunction with the planet (L3). A visual representation of the potential well and Lagrangian points is at Figure 1. Objects with little energy to climb out of the potential well are trapped around the L4/L5 point on what is termed a tadpole orbit after the shape the libration takes. Brown [4] suggested the possibility of another orbit type, the horseshoe orbit, related to Lagrangian points. With sufficient energy to leave L4/L5, but insufficient to depart completely, the object oscillates around the potential well, reflecting off the potential well's wall as it nears the planet at each end of the cycle.

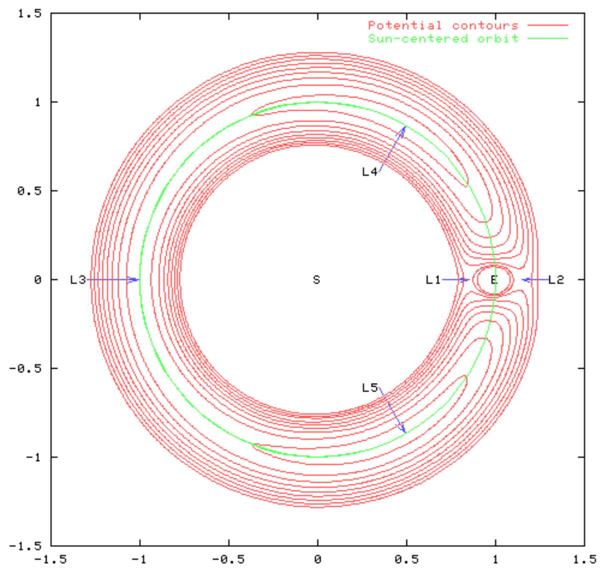


Figure 1: Earth-Sun Lagrangian (L1–L5) points and potential well. The plot's origin is the centre of mass of the Sun (S) and Earth (E). The contours can be considered the boundaries of motion for effectively massless particles with given energy.

Asteroid populations (Trojans) have been found librating on tadpole orbits around the L4 and L5 points of Jupiter, the first by M. Wolf in 1906. In 2001 there were only two known examples of horseshoe orbits: between the saturnian satellites Janus and Epimethius [5], and between asteroid 3753 Cruithne (1986TO) and Earth [6]. While the dynamic motion of 3753 Cruithne is of interest, it's high eccentricity and inclination ($e \simeq 0.515$, $i \simeq 19.8°$) make the orbit substantially different from that of Earth.

Discovery of the near-Earth asteroid 2002 AA29 by the LINEAR search (Stokes et al. [7]) was announced by Smalley et al. [8]. The original orbital parameters were refined by Block, Scotti, and Marsden [9]. The orbit is very similar to that of Earth, with the only major difference being the asteroid's 10.7° inclination. The orbital evolution of 2002 AA29 has been discussed by Connors et al. [1] who determined that the asteroid is in a horseshoe orbit relative to Earth. During the horseshoe phase, 2002 AA29 appears to spiral around the orbital path of the Earth, cycling from a leading to trailing position over a period of approximately 95 years. Connors et al. also determined that there are periodic transitions into an Earth quasi-satellite state in which 2002 AA29 stays within 0.2 AU of Earth for several decades. During the quasi-satellite phase the asteroid occupies the region normally excluded by its horseshoe motion.

3 Method

This investigation will be performed using the Swinburne SWIFT solar system simulator over six weeks in a number of phases as outlined below.

- Preparatory work including solar system state determination, test simulations, development of data transformation routines to convert from the simulator output to a reference frame co-rotating with Earth, and plotting utilities.

- Simulate the orbit of 2002 AA29 in a realistic solar system environment over a period of approximately 200 years. The aim of this simulation is to establish the presence of the baseline horseshoe orbit and provide data for the visualisation of that orbit. The integration step will be kept small ($\sim$ 2 days), and data output will be in steps of not more than 14 days.

- Extend the simulation forward for a period of approximately 1000 years in order to ascertain when the asteroid enters the quasi-satellite state indicated by Connors et al. [1]. Once again the integration time step will be kept small but data output will be less frequent at approximately two monthly intervals. Plots of Earth–2002 AA29 distance should show an atypical minimum in the region of the quasi-satellite phase before returning to the oscillatory horseshoe cycle.

- In the event that quasi-satellite behaviour is identified, higher resolution analysis of the region surrounding the transition to and from this state will be undertaken. The data analysis will compare this behavioural change to the baseline horseshoe behaviour identified earlier. In particular, changes in eccentricity and semi-major axis will be examined.

- The simulations above will be repeated with only the terrestrial planets present. The aim of these simulations is to discern if the presence of the gas giants, particularly Jupiter, has any impact on the medium term evolution of 2002 AA29's orbit.

The initial solar system state will be determined using the NASA JPL Horizons system to provide Sun-centred range and velocity vectors for the epoch at which the most recent Minor Planet Electronic Circular 2002 AA29 orbital determination was made. The asteroid will be treated as a massless test particle for the purposes of simulation.

Time permitting, secondary simulations will be performed to determine the sensitivity of the asteroid orbit simulation to initial parameters. In these simulations the initial solar system state will be determined by calculation based on base positions and corrections from Standish et al. [10] as reproduced by NASA JPL Solar System Dynamics Group [11].

4 Simulation Runs

The simulator being used is the SWIFT system written by Martin Duncan and Hal Levison [12, 13] using the regularised mixed variable symplectic integrator. User input and job control for the Swinburne supercomputer is achieved through a custom WWW interface.

	Data set	Planets	T_{int}	T_{step}	T_{out}	Repeats
1	Primary	Mercury	40	0.001	0.02	30
2	Primary	All	40	0.001	0.02	30
3	Primary	All + 18 Test particles	2000	0.001	1.00	0
4	Primary	Terrestrial	40	0.001	0.02	30
5	Primary	All	2000	0.001	1.00	5
6	Primary	Terrestrial	2000	0.001	1.00	5
7	Primary	All	2000	0.00027	1.00	0
8	Secondary	All	40	0.001	0.02	30

Table 1: Simulation summary.

A set of 8 simulations were run using solar system state and simulator parameters discussed in Section 4.1. Asteroid 2002 AA29 was treated as a massless particle in all simulations. The simulations and associated parameters are outlined in Table 1.

Simulation 1 was performed to validate the choice of $T_{step} = 0.001$. In the absence of other gravitating bodies the orbit of Mercury, the fastest and most curved of the planet orbits, about the Sun should be an ellipse with invariant characteristics. Simulator output shows no obvious visual changes in the orbit. Variations in output orbital elements were only evident at the least significant digit. The variations are small enough to ignore, so for the purposes of this project the chosen integration time step is adequate.

Simulation 2 is the primary source of detailed data for this project. Its purpose is to establish the baseline horseshoe and, if present, the quasi-satellite behaviour of the asteroid's orbit. Data output from this simulation was post-processed to convert it into an Earth co-rotating frame.

Simulation 3 investigates the sensitivity of the asteroid behaviour to the location of the asteroid's nodes. The eighteen test particles are set up identically to 2002 AA29 except that the longitude of the ascending node is varied nine degrees either side in one degree increments.

Simulation 5 aimed to determined if the behaviours identified in Simulation 2 persisted over a longer period.

Simulations 4 and 6 repeated Simulations 2 and 5 without the presence of the outer planets. The aim of these simulation was to determine if these planets substantially affected the behaviour of 2002 AA29.

Simulation 7 and 8 aim to identify the sensitivity of outcomes to the choice of integration time step and the initial positioning of the planets.

4.1 Solar System State

Initial solar system state was determined using two different methods for the planets with a single method for 2002 AA29:

- For asteroid 2002 AA29 the best available published orbital elements were calculated at 22 November 2002 (JD 2452600.5) (Block et al.[9]). These elements were used unaltered.

- The NASA JPL Horizons system was used to provide the Sun-centred position and velocity vectors for the barycentres of the nine major planets at the epoch. Velocity vectors were converted from AU/day to AU/year by multiplying by 365.25 days/Julian year. Mean motion was converted from degrees/day to degrees/year. These values are used as the primary planetary figures.

- Using tables from Standish et al. [10] as reproduced by NASA JPL [11] to determine a secondary set of planetary orbital elements. The tables give base orbital elements in J2000 coordinates for the epoch

(JD 2451545.0) and linear corrections quoted per Julian century. Accuracy is expected to better 20" for the terrestrial planets but errs by up to 600" for Saturn. Pluto is not accurately modelled.

Planetary orbital information was calculated for the same epoch as the orbital solution for asteroid 2002 AA29.

The Horizons system provided Cartesian coordinates directly, with only a simple unit conversion required for the velocities. For the Kepler elements a conversion to Cartesian position and velocity vectors was achieved by applying the following steps:

- Compute the eccentric anomaly. This involves solving the transcendental Kepler's equation, $M = E - e\sin E$, for E. As speed was not an issue in this conversion a simple implementation of the Newton-Raphson method was used to solve for E. All angular measures are expressed in radians.

- The object's true anomaly, ν, is derived from the eccentric anomaly using the relation:

$$\nu = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\right)\tan\frac{E}{2}$$

- Calculate the object radius from the primary focus of its orbit (Sun) using $r = a(1 - e\cos E)$.

- Calculate the mean motion of the object. Since the units used are AU and years the period of the orbit is $P = \sqrt{a^3}$, and $n = 2\pi/P$. Units for mean motion are generally quoted in degrees per day but the calculation here is in radians per year to maintain compatible units throughout.

- The object's Cartesian position and velocity vectors are calculated in a two-dimensional system in the plane of the orbit, centred on the primary focus with positive X in the direction of perihelion and positive Y 90 degrees away in the direction of the orbit. The equations are:

$$\begin{aligned} \mathbf{r} &= r[\cos\nu, \sin\nu] \\ \dot{\mathbf{r}} &= \frac{na}{\sqrt{1-e^2}}[-\sin\nu, e + \cos\nu] \end{aligned}$$

- The two dimensional solution must be transformed into the final three dimensional (XYZ) system. This is achieved by applying a series of

3D rotations: ω about the Z axis ($P1$), I about the Y axis ($P2$), and Ω about the Z axis ($P3$). Represented as a series of matrix operations:

$$P_1 = \begin{bmatrix} \cos\nu & -\sin\nu & 0 \\ \sin\nu & \cos\nu & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos I & -\sin I \\ 0 & \sin I & cos I \end{bmatrix}$$

$$P_3 = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R} = P_3 P_2 P_1 \mathbf{r} \qquad \dot{\mathbf{R}} = P_3 P_2 P_1 \dot{\mathbf{r}}$$

Appendix A shows the planetary masses, Kepler elements, and Cartesian vectors used to initialise the simulations.

4.2 Co-rotating Frame

The restricted three-body problem that underpins the derivation of Lagrangian points and the existence of tadpole and horseshoe orbits is for co-planar circular orbits. The frame of reference rotates about the centre of mass at a constant rate so that the line connecting the primary and secondary masses forms the primary axis.. This arrangement is depicted graphically in Figure 2. The situation being simulated for this project is not

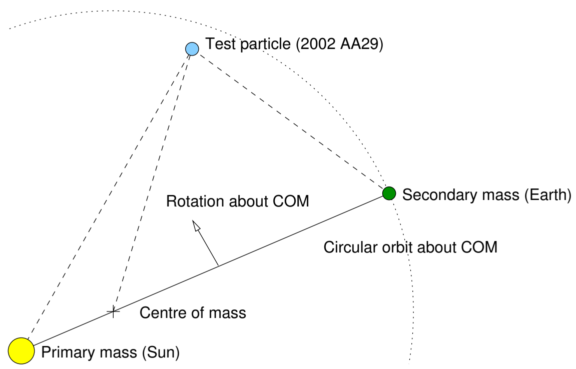


Figure 2: The co-rotating frame of reference for the derivation of Lagrangian points, and horseshoe and tadpole orbits.

a perfect match to the theoretical system. The Earth and asteroid orbits:

- Are not coplanar. Excursions above and below the ecliptic plane caused by inclination to the ecliptic could be removed by rotating the orbital planes back into the ecliptic, but a reasonable approximation is obtained by simply ignoring the out-of-plane component of orbital position vectors (working in X-Y coordinates only).

- Are not circular. The eccentricity of both orbits is low so, to a reasonable approximation, they can be treated as circular.
- Are not traversed at uniform angular velocity. The angular velocity of the Earth along its eccentric orbit,$e \simeq 0.0167$, will not be uniform, as in theory, but the variation is not substantial. The angular velocity is given by:

$$\dot{\nu} = \frac{na^2\sqrt{1-e^2}}{r^2}$$

At perihelion $r = a(1-e)$, and $\dot{\nu}$ is maximum, while at aphelion $r = a(1+e)$ and $\dot{\nu}$ is minimum. The ratio of maximum to minimum $\dot{\nu}$ is:

$$\frac{\dot{\nu}_{perihelion}}{\dot{\nu}_{aphelion}} = \frac{(1+e)^2}{(1-e)^2} = 1.069$$

The seven percent variation around mean motion should not substantially distort the conversion into a co-rotating frame.

For the Sun-Earth system the mass ratio is in the order of 300000:1. The centre of mass of this system is so close to the centre of the Sun that for the purposes of this analysis they will be considered collocated. This assumption simplifies conversion of Sun-centred simulator output to a co-rotating frame by allowing it to be achieved with a single rotation about the Z axis. At each time step a rotation is calculated to bring Earth back onto the positive X axis. The same rotation is applied to the asteroid position in order to maintain relative location and distance. Earth-asteroid range and Earth-Sun-asteroid angle were calculated for plotting purposes.

5 Analysis and Discussion

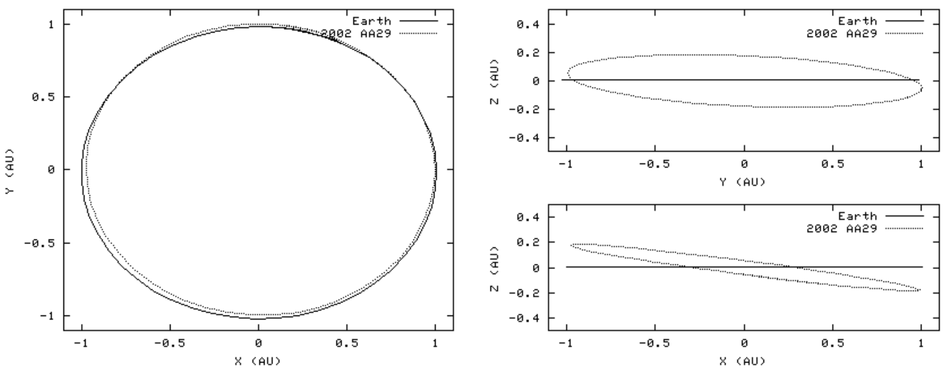
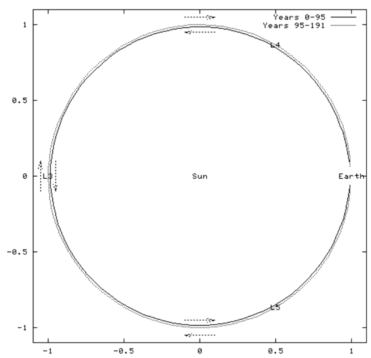


Figure 3: Earth and 2002 AA29 orbits looking along each major axis.

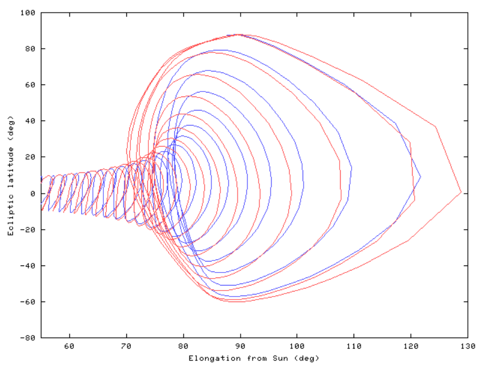
Figure 3 shows the orbit of the Earth and 2002 AA29 over the first year of simulation looking along each of the primary axes. The inclination of the asteroid orbit is clearly visible, as is the near perfect match in shape with Earth's orbit. Unless otherwise noted, all analysis has been performed using the data from Simulation 2.

5.1 Horseshoe Behaviour

Conversion into the Earth co-rotating frame makes the reason 2002 AA29's orbit is called a horseshoe quite plain. Figure 4(a) is a plot of 2002 AA29's annual average location relative to Earth for the first 192 years of the simulation. The asteroid position at time 0 is highlighted, Earth remains between the horns of the horseshoe, and the Sun is the centre of the plot. Over the first half of the plot period the asteroid creeps slowly anti-clockwise around the inner portion of the trajectory before approaching Earth from behind. The close approach reverses the average motion, which proceeds clockwise around the outer portion of the trajectory. At the end of the plot period the asteroid once again approaches Earth, reverses, and the cycle repeats. The apparent motion of the asteroid over a year is a circuit of the Earth's orbit. Figure 4(b) show the close approach near year 2193 AD as it would appear in the morning sky, with blue for motion toward Earth, and red for receding motion.



(a) Annual average position of 2002 AA29 in co-rotating frame for first 192 years of simulation



(b) Close approach near 2193 AD as seen in the morning sky.

Figure 4: Horseshoe behaviour.

The horseshoe behaviour manifests as a cyclical change in the asteroid's semi-major axis (SMA). Figure 5 shows the asteroid's annual average SMA over the entire simulation period. Close approaches to Earth, which occur at each near-vertical transition, perturb the asteroid's SMA. If the asteroid SMA is larger than Earth's, the asteroid appears to approach Earth from the leading side, and the close approach decreases the SMA. If the asteroid's

SMA is less than Earth's, the asteroid moves anti-clockwise around the inner horseshoe track, eventually approaches Earth from behind, and the SMA is increased by the close approach. Both eccentricity and inclination are also affected by close approaches to Earth, but do not have the same cyclic nature as the SMA.

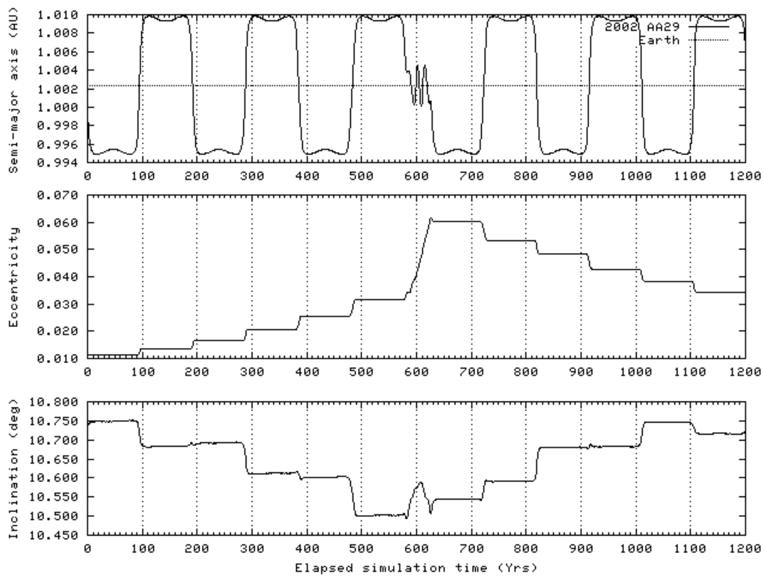


Figure 5: 2002 AA29 semi-major axis, eccentricity, and inclination versus time. The cyclical nature of SMA variation is characteristic of a horseshoe orbit; the irregularity around year 600 is the quasi-satellite phase.

An alternate representation of the horseshoe orbit, Figure 6, clearly shows the relationship between the asteroid's semi-major axis and the location of the Earth-Sun Lagrange points (L3, L4, and L5). The figure shows 194 years of the simulation with year 0 on the left. 2002 AA29 follows the trajectory anti-clockwise returning close to its original position 194 years later. The difference between the semi-major axis on the inward versus outward motion of the asteroid is greatest near the L4 and L5 points, and minimal near the L3 point. This is the result of interactions with the peaks and saddles of the potential in these regions.

5.2 Quasi-satellite Behaviour

Clearly visible in Figure 5 is a period around year 600 in which the cyclical change of semi-major axis is interrupted by a period of faster oscillations over a smaller range. Figure 7 shows the horseshoe close approach near year 600 and, for comparison, the previous two approaches at years 385 and 483. In the period from 580 to 630 the asteroid stays close to Earth and becomes a quasi-satellite of Earth. It has an apparent annual orbit about Earth, but

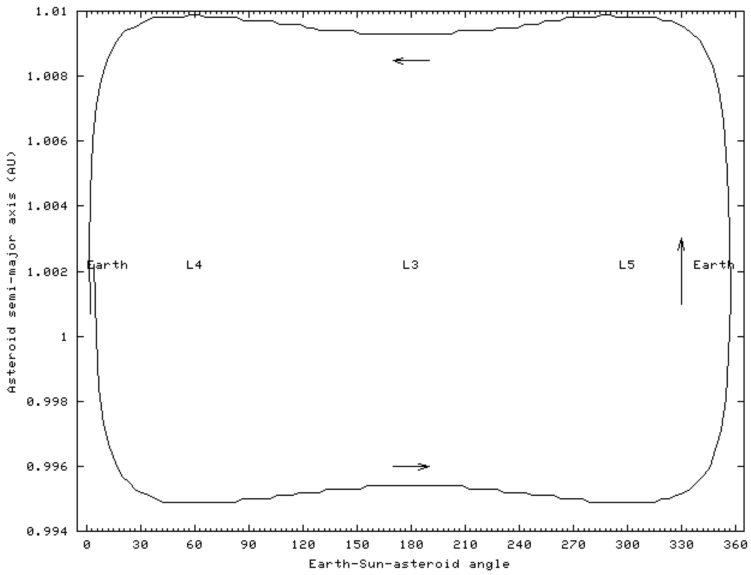


Figure 6: Semi-major axis versus location on the horseshoe expressed as an angle. The approximate location of the Lagrange points are marked.

the trajectory is not the ellipse of a true satellite, and the guiding centre of the orbit moves back and forth. Figure 8 expands the period of unusual SMA variation. The short period oscillation has peaks at 602 and 615 years, and troughs at 596 and 609. The 13 year period evident in this variation is close to 12:1 ratio with Jupiter's synodic period. The possibility that Jupiter is the trigger for, or provides the main driver of variation during, the quasi-satellite period will be investigated later. The eccentricity and inclination of the asteroid's orbit, while showing variation, are not cyclical within the quasi-satellite period.

Connors et al. [1] suggest that the 2002 AA29 quasi-satellite behaviour is a result of the degree of freedom added by the relatively large inclination. They further suggested that the motion is sensitive to the location of the test particle's nodes. This follows directly from the observation that close Earth-asteroid approaches can only occur near the nodes. Away from these regions, Earth asteroid distance is kept reasonably large by the asteroid's distance from the ecliptic. With this in mind the location of ascending and descending nodes was calculated by approximating the Earth's orbital plane with the ecliptic. Linear interpolation between simulator output steps either side of the XY plane was used to find the crossing point. A plot of the longitude of the ascending node (Ω) over the simulation period is at Figure 9. The ascending node is moving slowly clockwise over time ($\sim 0.2°/\text{Cy}$). During each close approach and the quasi-satellite period the value changes more dramatically ($\sim 1.1°/\text{Cy}$) before resuming a slow progression. The large

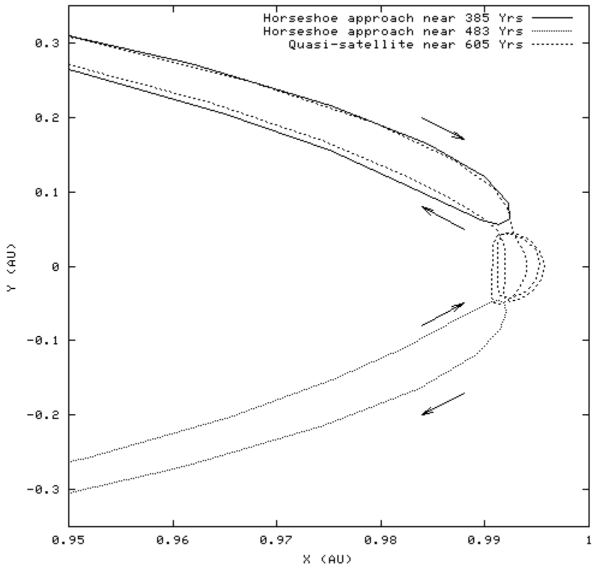


Figure 7: Three horseshoe close approaches. The approach near 600 years elapsed leads to quasi-satellite behaviour.

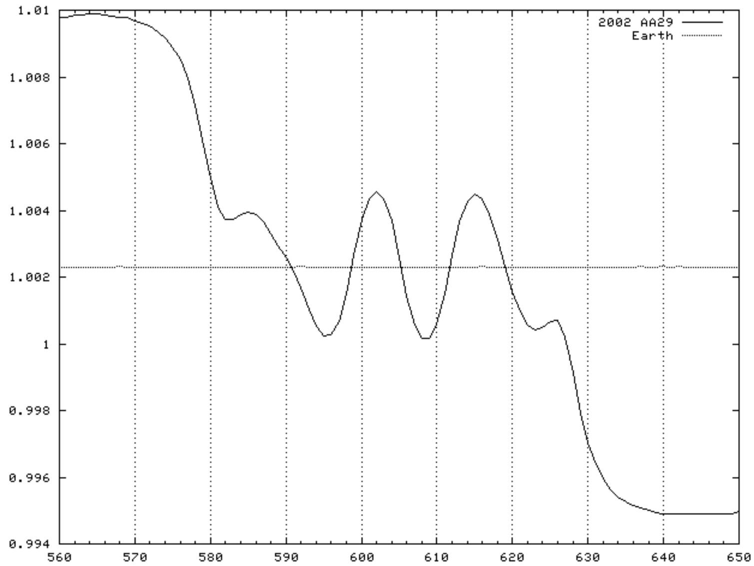


Figure 8: Semi-major axis variation during quasi-satellite phase.

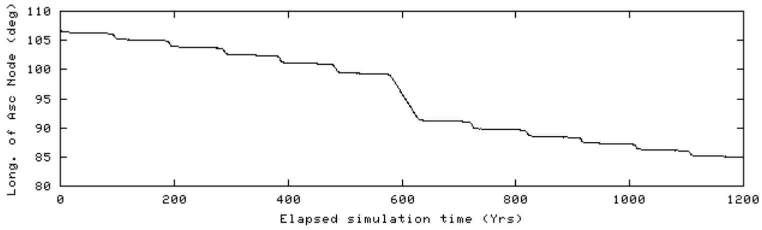


Figure 9: Longitude of the ascending node for 2002 AA29 over the simulation period.

change is effected by repeated close approaches to Earth. Simulation 3 runs 2000 years using 18 asteroid test particles with their longitude of the ascending node adjusted in steps of 1° between $\pm 9^\circ$ of the 2002 AA29 value. The separation in test particle Ω was chosen to be in the same order as the natural variation of 2002 AA29's Ω. All other test particle parameters remained identical to 2002 AA29. This spread has the effect of moving the location of the test particle's nodes and allows a rudimentary test of Connors et al. statement. Most of the eighteen test particles displayed some horseshoe behaviour, but generally only a few cycles. The test particles at $\pm 1^\circ$, -6°, -7°, and -8° displayed quasi-satellite behaviour during the simulation. The longest quasi-satellite period, Figure 10, shows the same cyclic behaviour noted earlier but with an approximate 16.8 year period.

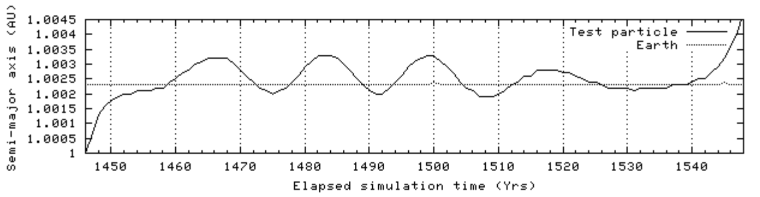


Figure 10: Semi-major axis variation during quasi-satellite phase for test particle started with 8° lower Ω than 2002 AA29.

For Jupiter to be causal trigger for the quasi-satellite phase it must provide an perturbation to the asteroid at the right point in its orbit. Given that Earth-asteroid close approaches can only occur near the asteroid's nodes these effects would be likely to occur near the line of nodes. Investigation of the location of Jupiter in relation to 2002 AA29's line of nodes shows no consistent alignment near the entry or exit of each quasi-satellite phase. Jupiter is sometimes leading and sometimes lagging the asteroid by varying amounts at these critical times. Similar alignments can be found near Earth-asteroid close approaches that do not enter quasi-satellite phase. Further, there seems no simple ratio between Jupiter's orbital period and the difference between onset dates, or between entry and exit dates, for the quasi-satellite periods as would expected if this were resonant behaviour. It seems, therefore, that the quasi-satellite periods are not triggered by Jovian gravitational effects.

The varied period of oscillation in a during the quasi-satellite phase seen in the simulations with varied Ω warranted closer scrutiny. An unmodified 2002 AA29 orbit over a period of 10000 years (Simulation 5) shows quasi-satellite behaviour starting at 580, 1780, 4050, and 4750 years elapsed. The semi-major axis oscillation period is 13, 13, 13.3, and 14.7 years respectively. The SMA oscillation is a very small range sinusoid, far smaller than the horseshoe oscillation, about that of Earth. The varying period does not support the hypothesis that the oscillatory behaviour is resonant with Jupiter. The trajectory shown in Figure 7 shows clearly that the asteroid oscillates between leading and trailing positions with respect to Earth during the year 580 quasi-satellite phase. This, coupled with SMA symmetry about Earth's SMA indicates that the behaviour is more likely to be the result of periodic interactions with Earth as the bodies pass each other near AA29's nodes.

While the asteroid behaviour in the quasi-satellite stage does not appear resonant with Jupiter, the possibility that the outer planets do affect the system's evolution bears investigation. The simulation was repeated without the outer planets (Simulation 4) with the expectation that behaviour would change slightly or that the quasi-satellite phase would disappear. The outcome for 2002 AA29 is different under these conditions as shown in Figure 11. The horseshoe behaviour persists until the close approach near year 600. However, rather than entering the quasi-satellite phase the asteroid's trajectory carries it straight past Earth continuing its apparent retrograde motion. The semi-major axis remains higher than Earth's for the remainder of the simulation period, and the horseshoe behaviour is broken.

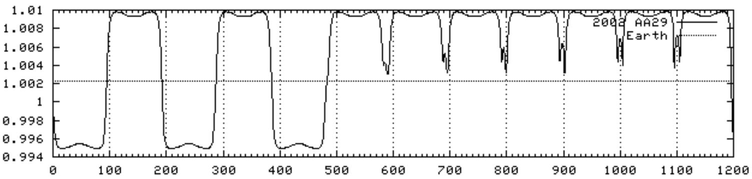


Figure 11: Semi-major axis for 2002 AA29 in the absence of the outer planets.

5.3 Longer Term Evolution

Looking at the longer term changes in semi-major axis, with and without the outer planets, was the subject of Simulation 5 and 6. The results are displayed at Figure 12. The horseshoe behaviour of AA29 appears to be a temporarily stable state that disappears around year 6230 with Jupiter present but only lasts until year 500 without the presence of Jupiter. The orbital eccentricity (not displayed) of the asteroid is constrained within a narrow, near-circular range with Jupiter present, but rapidly moves toward $e = 0.2$ without. The asteroid's orbital inclination (not displayed) declines steeply toward zero without Jupiter but suffers much more modest changes in the full solar system. Earth's orbital parameters are not substantially altered over the 10000 year period. Clearly the presence of the outer planets plays a part in the evolution of the inner solar system, in particular the asteroid-sized bodies. The outer planets appear to provide a calming effect on orbital behaviour in the inner solar system. Given that the majority of outer solar system mass is concentrated in Jupiter, it seems reasonable to attribute most of the effect to that planet.

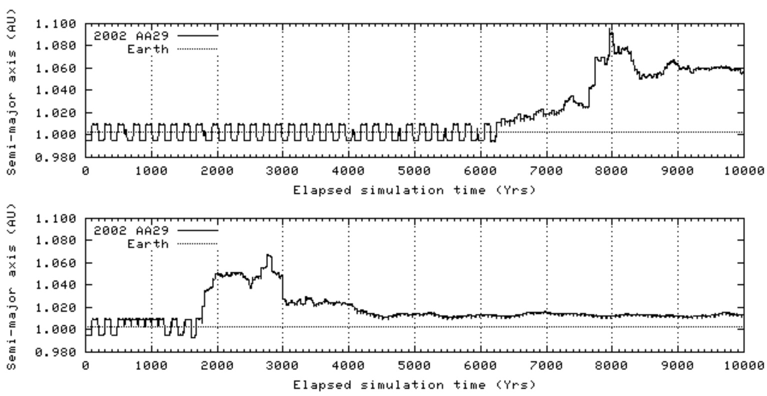


Figure 12: 2002 AA29 semi-major axis over 10000 years, with (top) and without the presence of the outer planets.

5.4 Sensitivity to Simulation Parameters

Several differences were noted between the results of Connors et al. [1] and the results of the simulations for this project. The first quasi-satellite period appears in both results. However, the second quasi-satellite phase, at circa 1780 years elapsed, occurs in Connors' results half a horseshoe cycle later. Later behaviour diverges from that seen in the present simulations. There are several potential causes for such a difference:

- Different asteroid starting state. Connors et al. used the first available orbital determination for 2002 AA29 [8] where this project uses a later determination [9].

- Different solar system starting point. This project uses the NASA JPL Horizons system to provide the initial locations for the planets.

Connors et al. used the JPL planetary ephemeris DE406 and included the effects of the Moon, Ceres, Pallas, and Vesta.

- The simulator algorithms differ. Of particular importance is the handling of the close approaches in this system.
- Simulation parameters, particularly integration time step, differ.

Connors et al. [1] chose to simulate at the interval of 0.1 days (0.00027 years) for determination of stability of the 2002 AA29 system. Connors' integration time step is a third of the time step used in this project's simulations. To verify that the behaviour of the system is not substantially altered by the choice of time step Simulation 7 was performed. The resulting plot of semi-major axis is at Figure 13. Coarse comparison with Figure 12 shows that the simulation gives similar results out to year 5000, after which the two simulations diverge. Adjusting the simulation time step does not align the current result with Connors et al., pointing at another cause for the difference in behaviour.

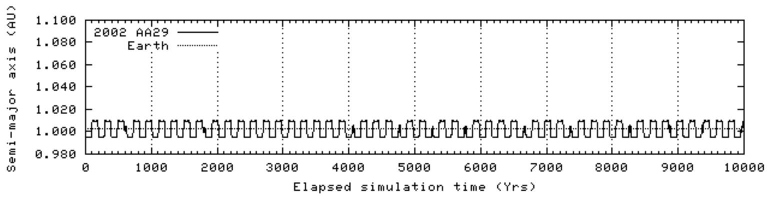


Figure 13: 2002 AA29 semi-major axis versus time for $dT = 0.00027$ years.

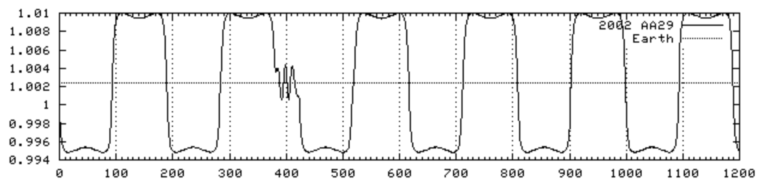


Figure 14: 2002 AA29 semi-major versus time for alternate solar system starting state

Simulation 8 was undertaken to determine if the simulation outcomes were sensitive to the precise positioning of planets. The alternate starting state for the solar system is derived from a widely used set of tables as described in Section 4.1. This alternate setup places Venus and Earth in a position closely approximating the primary setup. The other planets display varying degrees of difference to their primary counterparts. Pluto's orbit is grossly different. The resulting behaviour is shown in Figure 14. While the horseshoe behaviour remains, the quasi-satellite behaviour has moved. The simulation is sensitive to initial state of the solar system.

The simulations are sensitive to selection of integration time step in the longer term. The selection of solar system starting state, and presumably asteroid starting state, affects behaviour in the simulation over a short time frame. Consequently, the differences between the present results and those of Connors et al. most likely contain components of sensitivity to initial conditions.

6 Conclusion

The orbital dynamics of 2002 AA29 are interesting as an example of horseshoe interaction with Earth. The change out of horseshoe mode to quasi-satellite behaviour is unique among known examples of horseshoe orbits. The transition into quasi-satellite phase occurs several times in the simulated periods. Jupiter, and the outer planets, play a part in the evolution of the inner solar system but do not appear to be the trigger for entry or exit to the quasi-satellite stage. The asteroid's oscillatory motion about Earth seems to be purely the result of interactions with Earth near the asteroids nodes. The behaviour is sensitive to changes in the location of the asteroid's nodes, as indicated in Connors et al.[1].

Simulation of 2002 AA29's motion is sensitive to initial starting state for the solar system and asteroid, and to the integration time step. Reliability of the simulator output beyond the first two or three thousand years seems questionable.

Future work on the nature of the quasi-satellite phase could look in more detail at the precise nature of the nodal interactions between Earth and 2002 AA29.

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A Solar System State Information

Object	Mass	Object	Mass
Mercury	0.000173919	Saturn	0.29942066
Venus	0.0025643862	Uranus	0.045741297
Earth	0.0031464686	Neptune	0.053952704
Mars	0.00033807869	Pluto	6.8467899E-06
Jupiter	1.0	Sun	1047.0

Table A1: Masses for the major solar system objects. Units are $M_{Jupiter}$.

Object	a (AU)	e	i (Deg)	ω (Deg)	Ω (Deg)	M (Deg)
Asteroid Data						
2002 AA29	4.40299344e-01	8.98193842e-01	-1.29301215e-01	-5.51673841e+00	2.76485669e+00	8.49422854e-01
Secondary Solar System Data						
Mercury	3.87098949e-01	2.05631420e-01	7.00468128e+00	7.74610542e+01	4.83280874e+01	1.25925298e+02
Venus	7.23332017e-01	6.77180302e-03	3.39468704e+00	1.31532107e+02	7.66726877e+01	2.24823651e+02
Earth	1.00000011e+00	1.67091207e-02	-3.26798007e-04	1.02956809e+02	-1.14069623e+01	3.29224909e+02
Mars	1.52366022e+00	9.34157694e-02	1.85040555e+00	3.36053369e+02	4.95703507e+01	1.62946469e+02
Jupiter	5.20338056e+00	4.83889379e-02	1.30526669e+00	1.47605923e+01	1.00565920e+02	6.77594460e+00
Saturn	9.53698318e+00	5.41399765e-02	2.48450905e+00	9.24162958e+01	1.13702268e+02	-1.20845996e+02
Uranus	1.91913079e+01	4.71621760e-02	7.69843223e-01	1.70974776e+02	7.42163830e+01	8.04234011e+01
Neptune	3.00689273e+01	8.58659534e-03	1.76914078e+00	4.49645716e+01	1.31720476e+02	1.34507988e+02
Pluto	3.94816645e+01	2.48809528e-01	1.71418389e+01	2.24065698e+02	1.10303170e+02	-9.12438430e+01

Table A2: Orbital Elements for Asteroid 2002 AA29 and planets. All elements in J2000.0 coordinates at 2002 Nov 22.0 (JD 2452600.5).

Object	x (AU)	y (AU)	z (AU)	$\dot{x}$ (AU/Yr)	$\dot{y}$ (AU/Yr)	$\dot{z}$ (AU/Yr)
Asteroid						
2002 AA29	4.40299344e-01	8.98193842e-01	-1.29301215e-01	-5.51673841e+00	2.76485669e+00	8.49422854e-01
Primary Solar System Data						
Mercury	-1.32872000e-01	-4.46435866e-01	-2.42756580e-02	7.78580159e+00	-2.41737300e+00	-9.12042259e-01
Venus	2.18312668e-01	6.86947441e-01	-3.20638965e-03	-7.06517873e+00	2.20236074e+00	4.37921019e-01
Earth	5.01779981e-01	8.50821057e-01	-6.47402135e-06	-5.51499190e+00	3.16852503e+00	-3.03552975e-05
Mars	-1.64403678e+00	-9.59567891e-02	3.83828267e-02	4.89034233e-01	-4.66572028e+00	-1.09760378e-01
Jupiter	-3.20264274e+00	4.20803436e+00	5.41959012e-02	-2.22968154e+00	-1.54074226e+00	5.62928445e-02
Saturn	8.50257805e-01	8.99481010e+00	-1.90304669e-01	-2.13963444e+00	1.86215420e-01	8.18563857e-02
Uranus	1.69380802e+01	-1.06541827e+01	-2.59083795e-01	7.51882001e-01	1.14766018e+00	-5.43039997e-03
Neptune	1.94249483e+01	-2.29747044e+01	2.55246750e-02	8.65431179e-01	7.45528503e-01	-3.53104446e-02
Pluto	-6.62894832e+00	-2.94192958e+01	5.06573208e+00	1.13916677e+00	-4.48918395e-01	-2.82320138e-01
Secondary Solar System Data						
Mercury	-1.65207200e-02	-4.41085761e-01	-3.45161550e-02	8.55046472e+00	-1.55212747e+00	-9.11534933e-01
Venus	2.18654224e-01	6.93146141e-01	-3.14302056e-03	-7.00920255e+00	2.17640953e+00	4.34333340e-01
Earth	4.96174931e-01	8.51734486e-01	-5.32179292e-06	-5.53486174e+00	3.16032865e+00	-1.14258699e-05
Mars	-1.62761585e+00	-3.28378769e-01	3.31464804e-02	8.01877912e-01	-4.58000675e+00	-1.15677762e-01
Jupiter	-2.68277415e+00	4.16394255e+00	4.26938519e-02	-2.43800037e+00	-1.55075515e+00	6.10874217e-02
Saturn	1.67854038e+00	9.67411196e+00	-2.35425298e-01	-1.95739956e+00	2.50790992e-01	7.33930949e-02
Uranus	1.66866048e+01	-9.25435516e+00	-2.49589996e-01	7.57803564e-01	1.22728453e+00	-5.31307992e-03
Neptune	2.01984635e+01	-2.25199325e+01	-2.75484104e-03	8.52053622e-01	7.54911520e-01	-3.51608368e-02
Pluto	-3.23680877e+01	-2.40028969e+01	1.19322225e+01	6.96873461e-01	-6.12111526e-01	-1.36077868e-01

Table A3: Cartesian position and velocity vectors.